Multiple Causation, Apportionment and the Shapley Value

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ABSTRACT[1]

Multiple causation is one of the most intricate issues in contemporary tort law. Sharing a loss suffered by a victim among multiple tortfeasors is indeed difficult and courts do not always follow clear and consistent principles. Here, we argue that the axiomatic approach provided by the theory of cooperative games can be used to clarify that issue. We have considered the question from a purely theoretic point of view in Dehez and Ferey (2013). Here we propose to analyze it from a legal perspective. We consider the specific case of successive causation by defining and solving a general class of games – called "sequential liability games". We show that our model rationalizes the two-step process proposed by the *Restatement Third of Torts*, apportionment by causation and by responsibility. More precisely, we show that the weighted Shapley value associated to a sequential liability game is the legal counterpart of this two-step process.

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Outline

1. INTRODUCTION

"*Logic has not always the last word in law*"

Chief Justice of the New Hampshire Supreme Court Robert Peaslee (1934, p. 1131)

Multiple causation is one of the most intricate issues in contemporary tort law which arises when several tortfeasors cause harm to a victim entitled to recover it: courts have to apportion damages among them[2]. Many models and theories have been proposed in law[3], philosophy, economics[4], psychology[5] to capture the features of legal causation and apportionment issues and legal debates lead the American Law Institute to recently promulgate a new *Restatement* dedicated to this issue[6].

The paper adds to this literature by developing a cooperative game approach in which damages are monetized and modeled as cooperative games where players are the tortfeasors who jointly created an indivisible economic loss to be paid, the damage.[7] Solution concepts are then applied following an axiomatic method. Contrary to law and economics models in the literature, we are interested in the fairness of the apportionment rather than in the incentives created by the apportionment rules. Therefore we consider causation from an *ex post* perspective – once the damage occurred – and not from an *ex ante* perspective.[8]

[2] Four multiple causation issues may be distinguished: first is the successive causation; second, the simultaneous causation; third, the alternative causation and fourth, the victim's contribution.

[3] See Herbert L.A. Hart and Tony Honoré (1985) and John Borgo (1979) for a comprehensive analysis of causation related to the theory of law and Jules L. Coleman (1992) for causation issues related to moral theory. See also Richard Wright (1985), W. Page Keeton (General Editor), Dan B. Dobbs, Robert E. Keeton, and David G. Owen (1984) for specific issues of causation in Tort.

[4] In economics, a constant attention has been devoted to this topic. See William M. Landes and Richard A. Posner (1980); Mario J. Rizzo and Franck S. Arnold (1980), (1986); Steven Shavell (1983); Lewis Kornhauser, and Richard L. Revesz (1989); Francesco Parisi and Ram Singh, (2010).

[5] For a psychological approach on causation in law, see Jeffrey J. Rachlinski (1998) and more generally the literature about the hindsight bias in behavioral law and economics.

[6] See the *Restatement (Third) of Torts: Apportionment of Liability* promulgated by the American Law Institute in 1999 and published in 2000, and notably "Topic 5: Apportionment Of Liability When Damages Can Be Divided By Causation, § 26" (thereinafter, the *Restatement*).

[7] For a comprehensive view of the economics of causation, see Omri Ben-Shahar (2000). Surprisingly enough, the theory of cooperative game and its solution concepts have never been elaborated in the law and economics literature to analyze multiple causation issues. To our knowledge, and except an unpublished paper mentioned by Ben-Shahar (2000), no model of multiple causation cases is available in terms of cooperative game. See also the approach proposed by Matthew Braham and Martin van Hees (2009) which analyzes the concept of "the degrees of causal contribution for actual events" by using power indices.

[8] The model is more interested in retrospective causation than in prospective causation. According to Ben-Shahar (2000, p. 647) "Retrospective causation exists if, all else held fixed, but for the action the harmful consequence would not have occurred. Prospective causation exists when an action raises the probability of the harmful

In the following, we distinguish with Posner and Landes (1980) successive joint tort and simultaneous joint tort and we focus on a subset of multiple causation cases for the clarity of the exposition: the successive injury. Successive injury occurs when, after an injury caused by a first tortfeasor A to a victim V, the damage is aggravated by tortious acts from a second wrongdoer B, then from a third one C *etc*. A, B, C… are said to be the multiple tortfeasors because they cause *together* the final damage suffered by V. An example from the *Restatement* may illustrate such a case: suppose "A negligently parks his automobile in a dangerous location. B negligently crashes his automobile into A's automobile, damaging it. When B is standing in the road inspecting the damage, B is hit by C, causing personal injury to B. B sues A and C for personal injury and property damage. B's negligent driving and A's negligent parking caused damage to B's automobile. A's negligent parking, B's negligent driving, B's negligent standing in the road, and C's negligent driving caused B's personal injuries." (American Law Institute, 2000, Topic 5, §26, *comment c*).

How should judges determine the compensation to be paid by each injurer? Should he consider that the car driver A is liable for the entire damage insofar as without his action the damage would have not occurred? Or that each of them is liable for a part of it? Or that one of them is more liable than the other and by which amount? An apportionment rule is needed to correctly share the damage among them. Such litigations occur as soon as two or more individuals have jointly caused damages and it is easy to think about the different fields of law concerned by this issue: environmental law, nuisance, accident law, medical malpractices, products liability or even antitrust *etc.*[9]

In our model, *adjudication* specifies the compensation that each tortfeasor has to pay to the victim. *Adjudication* should be *unobjectionable* (Ferey and Dehez, 2013). There is a *minimum* compensation: each tortfeasors should pay at least the damage that he would have caused *alone*. There is also a *maximum* compensation: no tortfeasors should pay more than the *additional* damage that he has caused. The additional damage is measured by the difference

consequence. Thus, the distinguishing factor between the two types of causation is the time perspective of the evaluation. Retrospective causation is backward-looking, answering the counterfactual inquiry of whether the action was a necessary condition for the outcome. Prospective causation, in contrast, is forward-looking, answering the *ex ante* inquiry of whether the action increased the likelihood of injury". For a criticism of economic analysis of law related to *ex post* and *ex ante* perspectives on causation, see Wright (1985b). Our model escapes from this criticism and shows that economics has a lot to say on *ex post* causation.

[9] Three main approaches are distinguished in law : joint liability, several liability and joint and several liability. In joint liability, each tortfeasor is liable for the full amount of the damages, without any claim against the other tortfeasors. In several liability, each tortfeasor is only liable for a given share. In joint and several liability, each tortfeasor is liable for the total amount of damages but has a claim against the other tortfeasors to get their contribution to damage back. Therefore, sharing rules are needed in the last two cases.

between the total damage and the damage that would have resulted without the participation of that individual tortfeasor. These two inequalities can actually be found in tort law. Here we go further and extend them from individual tortfeasors to *subsets of tortfeasors*, leading to the following two conditions:

C1 *the contribution of any subset of tortfeasors should be at least equal to the damage they would have caused without the intervention of the others.*

C2 *the contribution of any subset of tortfeasors should not exceed the additional damage resulting from their participation.*

To apprehend the notion of unobjectionable adjudications, we construct a game with transferable utility – called (sequential) liability game – whose characteristic function reflects the *potential* damage caused by any subset of tortfeasors while capturing explicitly successive causation. We show that the *core* of a liability game defines the set of *all* possible unobjectionable adjudications and that the (symmetric) *Shapley value* defines an adjudication that is a fair compromise in which tortfeasors differ only in the damage they have caused. A judge may depart from that fair compromise by assigning weights to tortfeasors to reflect misconduct or negligence. The resulting *asymmetric* Shapley values define unobjectionable adjudications and, vice versa, weights can be associated to any unobjectionable adjudication.

Both legal practices and economic analysis of law are concerned by our analysis. First, our model provides a characterization of the apportionment rules at courts disposal. Second, we show that judicial practices, jurisprudence and legal debates underlie the solution concept that we use. For that purpose, we illustrate our model by some court decisions and by proposals and synthesis provided by the *Restatement*. We show that our approach offers a framework to better understand the two-steps process advocated by the *Restatement* based on apportionment by causation and apportionment by responsibility. Cooperative game theory is relevant for law and we aim at making judges and legal practitioners aware of the implicit logic they use to solve such cases. Moreover, discussing apportionment issues on the grounds of an axiomatic method may be useful to achieve greater fairness and greater consistency in adjudication.[10]

The remainder of the paper is organized as follows. In section 2, we define liability games and show that their core defines the set of unobjectionable adjudications. We show that it coincides with the set of weighted Shapley values. Section 3 deals with legal issues. We show

[10] As Coleman (1982, p. 349) asserts "political authority is necessary and inevitably coercitive […], exercising it requires a justification", and therefore "any body of the law must be coherent and consistent". See also Gerald Boston (1996, p. 269).

how the rule proposed by our cooperative game model enlightens the main legal principles and practices in tort law. We mainly rely on American common law cases on the one hand and on principles and proposals advocated by the *Restatement* on the other. We show how the two-step process proposed by the *Restatement* implicitly follows a cooperative game method and advocates apportionment rules which are equivalent to the core and the weighted Shapley value prescriptions. Section 4 concludes, followed by an Appendix that covers the essential game theoretic concepts used in the paper.

2. SEQUENTIAL LIABILITY GAMES

We consider *n* "players" who are involved in a damage to a person who is thereby entitled to compensation.[11] We focus more on *successive* causation than on simultaneous causation. Simultaneous means that the situation is so intricate that it is not possible to determine the sequence of acts that has resulted in the damage. Successive means *a contrario* that it is possible to determine that sequence and to assign an *additional* damage to each individual players.[12]

<u>Notation</u>: Lower-case letter will denote coalition sizes: $n = |N|$, $s = |S|$, $t = |T|, \ldots$ Coalitions $\{i,j,k,\ldots\}$ will sometime be written as $ijk\ldots$ $S \setminus i$ will denote the coalition obtained by *subtracting* player *i* from coalition *S*. It will also be convenient to define $x(S) = \sum_{i \in S} x_i$ for any vector $x \in \mathbb{R}^n$ and non-empty subset $S \subset N,$ with the convention $x(\emptyset) = 0.$

2.1 Liability sequences

In the successive case, the players are ordered, starting with player 1 who has caused the initial damage, subsequent players having caused successive aggravations. Each player is assigned the monetary value of the *additional* damage he has *directly* caused, $d_i \geq 0$ for player *i*. We do not exclude the case where $d_i = 0$ for some *i*. A player may indeed be part of a liability sequence without causing any direct damage. If a zero damage is assigned to a player located *at the end* of a liability sequence, that player is a *dummy* player. Hence, we assume that $d_n > 0$.

A liability situation is described by a set of players *N* and a damage vector $d = (d_1, \ldots, d_n)$. It is implicitly assumed that the natural ordering (1,...,*n*) defines the liability sequence. The

[11] We do not exclude the case where the victim herself is one of the player (victim's contribution cases). Her contribution is then simply deducted from the total damage to define the actual compensation received and the total compensation paid by the tortfeasors is then less than the damage.

[12] In what follows, we rely on the theory of cooperative games with transferable utility as exposed in the Appendix. References to results reported there are denoted by (Ax). See also Dehez and Ferey (2013) for details and proofs concerning liability games.

problem is to allocate the total damage among the n players, that means finding a vector $x = (x_1,...,x_n)$ such that:

$$x(N) = d(N)$$

where $d(N)$ is the total damage. In allocating the total damage among the players, one condition arises naturally: no player should pay for damage caused upstream i.e. each player should only be held responsible for the immediate and subsequent damages.

A transferable utility game (N,v) can be associated to any liability situation (N,d) where $v(S)$ measures the total damage the members of S are responsible for *together*, outside the intervention of the other players. It is the *potential* damage caused together by the members of S. Clearly we have $v(S) = 0$ if $1 \notin S$ and to compute $v(S) = 0$ if $1 \in S$, we only need to evaluate $v(S)$ for coalitions of the form $S = \{1,...,i\}$. Indeed, we have:

$$v(1,2,3,...,i) = \sum_{j=1}^{i} d_j \text{ and } v(S) = \sum_{j=1}^{i} d_j \text{ for all } S \supset \{1,2,...,i\} \text{ such that } i+1 \notin S$$

The resulting characteristic function v satisfies $v(N) = d(N)$ and $V(N \setminus i) = \sum_{j=1}^{i-1} d_j$. The 3-player liability game is defined by:

$$\begin{aligned}
&v(1) = v(13) = d_1 \\
&v(2) = v(3) = v(23) = 0 \\
&v(12) = d_1 + d_2 \\
&v(123) = d_1 + d_2 + d_3
\end{aligned}$$

Players i and j are *substitutable* in a liability game if player i and j are successive players and player i is assigned zero damage: $j = i+1$ and $d_i = 0$.

Liability games are *convex* and thereby *superadditive*. Liability games can be decomposed into sums of elementary games:

$$v(S) = \sum_{j=1}^{n} v_j(S) \tag{1}$$

where the elementary game (N,v_j) is defined by:

$$\begin{aligned}
v_j(S) &= d_j \text{ if } \{1,..., j\} \subset S \\
&= 0 \quad \text{otherwise}
\end{aligned} \tag{2}$$

The elementary game (N,v_j) can be written in terms of the *unanimity games* defined in (A3) i.e. $v_j(S) = d_j u_{T_j}(S)$. where $T_j = \{1,..., j\}$.

2.2 The core

The core (A6) of a transferable utility game is the set of allocations x such that $x(S) \geq v(S)$ for all $S \subset N$. Hence, by definition of $v(S)$, the core of a liability games defines allocations of the total damage such that no subset of players contributes *less* than its potential. This is condition C1 as specified in the Introduction. It implies that $x_1 \geq d_1$ and $x_i \geq 0$ for all $i \geq 2$. The first player always contributes an amount at least equal to the initial damage. By (A8), we know that core allocations satisfy the following inequalities:

$$x(S) \leq v(N) - v(N \backslash S) \text{ for all } S \subset N$$

This is condition C2 that imposes that no subset of players contributes more than its additional damage. Following (A8) the last inequalities reduces to

$$x_i \leq v(N) - v(N \setminus i) \text{ for all } i \in N$$

when applied to individual players. We notice that by definition of the core, C1 and C2 are actually equivalent conditions. Hence, core allocations are unobjectionable adjudications. Objections could indeed come from an individual tortfeasor who is asked to contribute more than the additional damage he has caused. Objections could also come from a group of tortfeasors who are together asked to contribute more than the additional damage they have *jointly* caused.

The core of a liability game can actually be defined by a limited number of inequalities:

$$C(N,d) = \left\{ x \in \mathbb{R}_+^n \,\middle|\, x(N) = d(N) \text{ and } \sum_{j=1}^{i} x_j \geq \sum_{j=1}^{i} d_j \text{ for all } i \in N \right\} \tag{3}$$

It contains the allocation $(d(N), 0, ..., 0)$ that imposes to the first player to pay the entire damage. It also contains the damage vector d that imposes to each player to pay exactly the additional damage he is directly responsible for.

A liability game is convex and, consequently, its core is the polyhedron whose vertices are the marginal contribution vectors as defined by (A4). For n = 3, there are four distinct vectors:

π	1	2	3
123	d_1	d_2	d_3
132	d_1	$d_2 + d_3$	0
213	$d_1 + d_2$	0	d_3
231	$d_1 + d_2 + d_3$	0	0
312	d_1	$d_2 + d_3$	0
321	$d_1 + d_2 + d_3$	0	0

In general, the core has 2^{n-1} vertices, one of which is the vector *d* itself. At all other vertices some players are exempted of contribution. Each exemption configuration is associated to one or more players' ordering though there is *one and only one* vector for each exemption configuration. For instance, in a six-player case, exempting players 2, 4 and 6 leads to the vector unique $(d_1 + d_2, 0, d_3 + d_4, 0, d_5 + d_6, 0)$. It is indeed the only vector consistent with the upstream condition.

2.3 The Shapley value

The Shapley value is defined in (A10) as the average of the *n*! marginal contribution vectors. Using the dummy player and symmetry axioms, we obtain the Shapley value of an elementary game (N, v_j) as defined in (2):

$$\begin{aligned} SV_i(N, v_j) &= \frac{d_j}{j} \quad \text{if } 1 \le i \le j \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

Using (1) and the axiom of additivity, we then obtain the Shapley value

$$SV_i(N, d) = \sum_{j=1}^{n} SV_i(N, v_j) = \sum_{j=i}^{n} \frac{d_j}{j}$$

$$SV_n(N, d) = \frac{d_n}{n}$$

$$SV_{n-1}(N, d) = \frac{d_n}{n} + \frac{d_{n-1}}{n-1}$$

$$\ldots$$

$$SV_2(N, d) = \frac{d_n}{n} + \frac{d_{n-1}}{n-1} + \ldots + \frac{d_2}{2}$$

$$SV_1(N, d) = \frac{d_n}{n} + \frac{d_{n-1}}{n-1} + \ldots + \frac{d_2}{2} + d_1$$

One recognizes the triangular formula of the Shapley value applied to the airport game introduced by Littlechild and Owen (1973).[13] We observe that the upstream condition is satisfied: what a player contributes does not depend on damage caused upstream. It is to be contrasted with the equal surplus rule (A5) that results in the following allocation:

[13] Liability games are indeed duals of airport games.

$$ES_1(N,d) = \frac{d_2 + ...d_n}{n} + d_1$$
$$ES_i(N,d) = \frac{d_2 + ...d_n}{n} \quad \text{for } i = 2,...,n$$

By convexity, the Shapley value defines an allocation that is centrally located in the core. It is therefore an unobjectionable adjudication that can be viewed as a fair compromise when there is no reason to treat the players differently. This can be viewed as the application of the principle of "insufficient reason".

A court may however consider that the players are not equally responsible. That possibility can be accommodated by using the weighted Shapley value (A11) with the possibility of a zero weight being assigned to some players. The symmetric Shapley value is then the particular case where weights are equal. The weighted Shapley value $SV(N,d,w)$ associated to given positive weights $(w_1,...,w_n)$ is given by:

$$SV_i(N,d,w) = \sum_{j=i}^{n} \frac{w_i}{w(T_j)} d_j \tag{4}$$

It is obtained by generalizing the argument used to define in the symmetric case, using the fact that the Shapley value of the elementary game (N, v_j) associated to positive weights *w* is given by:

$$\begin{aligned} SV_j(N, v_i, w) &= \frac{w_i}{w(T_j)} d_i \quad \text{if } 1 \le i \le j \\ &= 0 \qquad\qquad \text{otherwise} \end{aligned}$$

Again, we observe that the upstream condition is satisfied. The set of all weighted values is obtained by limit arguments, letting some but not all w_i go to zero. We know from (A12) that under convexity there is equivalence between core allocations and weighted values, i.e. weighted values are unobjectionable judgments and weights can be associated to any unobjectionable judgment. Using (4) it is easily seen that a *unique* set of normalized weights can be associated to any allocation *x* satisfying $x_1 + d_1$ and $x_i + 0$ for all $i \ge 2$.[14]

[14] Boundary allocations like the core vertices correspond instead to situations where some weights tend to zero at different velocities.

3. APPLYING THE WEIGHTED SHAPLEY VALUE TO THE LAW

Is our model meaningful for law? May the analytical framework developed in section 2 be useful for understanding and explaining legal practices and legal debates about apportionment of liability? This section addresses this issue. The "liability game" mathematically defined in the previous section is now considered from a legal point of view. We aim at showing that cooperative game theory and its solution concepts are relevant to improve our understanding of the common law, although it is not explicitly acknowledged by courts. In the following, we propose to rationalize the principles and methods advocated by *the Restatement* and analyze some legal cases which implicitly use a weighted Shapley scheme to apportion damage among multiple tortfeasors.

3.1. Cases and examples

The most illustrative examples of our model are the successive accident cases where the tortfeasors tortious acts are related.[15] But a lot of others cases and litigations are covered by our model: medical malpractices, enhanced injury, background conditions, victim's contribution and also some nuisances or product liability cases. In these issues, a same mathematical structure can be found as soon as the different tortfeasors (including the victim) follow a temporal chain of causality: in these cases, tortious acts of the tortfeasor i are a physical cause of "direct" damage d_i and a proximate or legal cause of the aggravated damages up the liability sequence (the enhanced injuries d_j with $j > i$). We provide further examples of these different kinds of litigation. Thereafter, all these cases will be named "successive injury cases".

Successive accident cases. In *Maddux*[16], the first tortfeasor hits the plaintiff's car and thirty second later, a second driver hits the car and caused other injury. The causal events are so close that the chain of injuries may be considered as a unitary event.

Background conditions. In *Steinhauser*[17], the Court had to adjudicate a case where the tortious act of the defendant had caused a "chronic schizophrenic reaction" from the plaintiff. The Court held that the defendants could explore the possibility of plaintiff having developed schizophrenia regardless of the accident.

[15] We exclude unrelated cases insofar as the second tortious act is not a legal cause of the damages up: apportionment is simple and is proportionate to each harm separately evaluated.

[16] *Maddux v. Donaldson*, 108 N.W.2d 33 (Mich. 1961).

[17] *Steinhauser v. Hertz Corp*., 421 F.2d 1169 (2d Cir.1970). *S*ee also *Lancaster v. Norfolk and Western railway Co.*, 773 F. 2d 807, 822 (7th Circuit. 1985).

Victim's contribution. In *Prospectus Alpha Navigation Co*[18], the plaintiff's ship was tied up at the defendant's dock. Because of a negligent tortious act of the plaintiff's crew, the ship caught fire. But the defendant was also negligent: he send the plaintiff's ship away before the fire being completely extinguished. Then, the fire caused further damage. In *Dillon*[19], a young boy was on a high beam of a bridge trestle. He lost his balance and was falling to the rocks when he grabbed the electric wires, negligently exposed by the defendant, which killed him.

Product liability. In *Hillrichs* [20], the Court considered that a jury could evaluate the extent of the enhanced injury. A corn harvesting machine was not equipped with an emergency stop device and the plaintiff lost his fingers after his hand had been entangled in. The Court considered that some evidence showed that the injury would have been different with a stop emergency device. In *Reed* [21], the plaintiff's was involved in a car accident in which the shattering of the fiberglass top of his car hurts his arm. The expert testified that such injury would have been avoided by a metal top. The Court considered that estimation of the enhanced damages was possible.

In all these situations, sharing the compensation due to the victim among several tortfeasors is required. But how should the courts proceed? One of the innovation proposed by the *Third Restatement* compared to the *First* or the *Second* is a "two-step process" to apportion damage among tortfeasors[22]. The method provides a unified framework taking account of the different issues: causation, degree of responsibility, divisibility, inconsistent verdicts *etc*. The method is stated as follow: "the factfinder divides divisible damages into their indivisible component parts. The factfinder then apportions liability for each indivisible component part under Topics 1-4. For each indivisible component part, the factfinder assigns a percentage of comparative responsibility to each party or other relevant person. See §§ 7, 8. The percentages of comparative responsibility for each component part add to 100 percent [...]. The plaintiff is entitled to judgment in an amount that aggregates the judgments for each component part".[23] By using both numerical examples and the literary explanation of the method, we show that this "two-step process" is equivalent to the weighted Shapley value of the corresponding sequential liability game.

[18] *Prospectus Alpha Navigation Co v. North Pacific Grain Growers*, Inc., 767 F. 2d 1379 (9th Cir. 1986).

[19] *Dillon v. Twin State Gas & Electric Co.*, 163 Atl. 1ll (N.H.1932).

[20] *Hillrichs v. Avco Corp.*, 478 N.W.2d 70 (Iowa 1991).

[21] *Reed v. Chrysler Corp.*, 494 N.W.2d 224 (Iowa 1992).

[22] Topic 5 of the *Restatement* is entitled "Apportionment Of Liability When Damages Can Be Divided By Causation". See also the *Restatement (second)* § 879 and Boston (1996).

[23] The *Restatement*, Topic 5, §26, *comment* c.

3.2. Divisibility, potential damages and characteristic function

The *Restatement* states that "damages can be divided by causation when any person or group of persons to whom the factfinder assigns a percentage of responsibility (or any tortious act of such a person) was a legal cause of less than the entire damages".[24] Damage is divisible when it is possible to assign to one tortfeasor the part of the damage he has caused alone but also when it is possible to divide the damage by subsets of tortfeasors.[25] On the contrary, damage is indivisible when several tortfeasors have jointly caused it without any possibility to know which part of the damage has been caused by each of them.

The characteristic function provides such a division of damage. Reciprocally, the factfinder or a jury instructed by a court to divide the damage seeking to assign to each subset of tortfeasors the damage they would have caused alone defines a characteristic function. Sometimes, the task is easy because the aggravated damages d_i are perfectly observable; sometimes, a counterfactual is needed. The factfinder wonders which amount of damage would have occurred if one of tortfeasors had not acted tortiously and defines a *potential damage*. Our model captures these features insofar as all the coalitions but the grand coalition are only hypothetical.[26]

If we analyze this method through our model, it is possible to divide damage by recurrence: tortfeasor 1 has caused a divisible damage for d_1 and the set of all tortfeasors (included tortfeasor 1) has caused an "indivisible" damage, $d(N) - d_1$. But $d(N) - d_1$ is also divisible: the set (1,2) has caused d_2 and the set of all the tortfeasors (included tortfeasor 1 and 2) has caused $d(N) - d_1 - d_2$ *etc*. Our model divides the full damage in several indivisible parts caused by different sets of tortfeasors. Therefore, damage is both divisible and indivisible which is the case dealt with by the Topic 5 of the *Restatement*. It is divisible since each of tortfeasor adds a marginal damage. However, the damage is also indivisible since the final damage $d(N)$ is the result of the joint behaviors of all tortfeasors. Such elements are not ignored by legal practitioners. Common law cases and the *Restatement* offer good examples.

[24] See the *Restatement*, Topic 5, §26, *comment* a: "Divisible damages can occur in a variety of circumstances. They can occur when one person caused all of the damages and another person caused only part of the damages. They can occur when the parties caused one part of the damages and nontortious conduct caused another part. They can occur when the nontortious conduct occurred before or after the parties' tortious conduct. They can occur in cases involving serial injuries, regardless of the length of time between the injuries. They can occur when the plaintiff's own conduct caused part of the damages." (Topic 5, §26, *comment* f).

[25] Interestingly enough, the *Restatement* mentions explicitly the "set" of tortfeasors: "Divisible damages may occur when a part of the damages was caused by one set of persons in an initial accident and was then later enhanced by a different set of persons" (the *Restatement*, Topic 5, *Reporters' note*, *comment* f).

[26] We rely on the classical distinction between prospective causation and retrospective one, see note 8 *supra*.

Some cases provide illustrative examples of the divisibility *vs* indivisibility issue. In the *Alpha Navigation* case, the court considered that a division of the full damage is needed: first is the part due to the fire itself and second is the part due to the defendant decision to send the ship away. Court had in mind the fact that subsequent tortfeasors have aggravated the damage[27] and that it was possible to measure the initial damage and the subsequent one. In case of medical malpractices, the physician is often held responsible for aggravating the damage caused first by a negligent tortfeasor. The same is held when a victim's pre-existing condition or disease exists which has made damages more important than he would have normally been. In the *Steinhauser* case[28], for example, the Court held that a division of damage is possible among the victim and the tortfeasor insofar as damage is larger because of the victim existing disease or prior accident (Keaton, 1984, p. 353).

From a general point of view, we can follow the *Restatement* by saying that "any enhanced injury case requires causal division between the plaintiff's original damages"[29]. The *Restatement* synthesizes ideas and practices by courts when they use the notion of potential damages. Potential damage is the damage which would have been caused by A if others tortfeasors had not been negligent.[30] Potential damage captures the fact that an agent has only aggravated the damages which would have occurred anyway. For example, in *Dillon*[31], Court used potential damages to drastically reduce the amount paid by the electric company by holding that even if the company had not been negligent, the boy would have suffered important damage due to his fall. The only damage the electric company has caused is, at most, the difference between actual damage and potential one. Commenting this case, Chief Justice Peaslee said that "serious injury, if not dead, was certain to ensue, when he was caught upon the defendant's wires and electrocuted" and therefore, it was fair and logical that Court allows damages for only such a sum "as his prospects for life and health were worth at the time the defendant's fault became causal" (Peaslee, 1934, pp. 1134-1135). In other words,

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[27] In *Lancaster v. Norfolk and Western railway Co.*, 773 F. 2d 807, 822 (7th Circuit. 1985), Judge Richard A. Posner develops the argument that a defendant may be entitled to an instruction directing the jury to reduce the plaintiff's damages by the probability that the plaintiff would have been injured notwithstanding the defendant's tortious act: "But a corollary to this principle is that the damages of the "eggshell skull" victim must be reduced to reflect the likelihood that he would have been injured anyway, from a nonliable cause, even if the defendant had not injured him". In *Evans v. United Arab Shipping Co.*, 790 F. Supp. 516, 519 (D. N.K. 1992), the District Court of New Jersey stated that "in cases like this one, in which an employer's negligence aggravated a preexisting condition, courts have held that the defendant must compensate plaintiff only for the aggravation itself and not for the preexisting condition" (see also Boston, 1996).

[28] *Steinhauser v. Hertz Corp.*, 421 F.2d 1169 (2d Cir.1970).

[29] The *Restatement*, Topic 5, §26, *Reporter's note*, *comment* a.

[30] In our model, potential damage is the value of the coalition of the subset of the tortfeasors involved.

[31] *Dillon v. Twin State Gas & Electric Co.*, 163 Atl. 1ll (N.H.1932).

Court divided the harm by evaluating the potential amount of damage due to the fall *alone*.[32] Similar legal reasoning could be found in other issues. For example, in *Douglas Burt & Buchanan Co*[33], the issue was to know whether the defendant had caused damage to the plaintiff by blocking the passage of a barge into a canal while the passage was already blocked by a landslide. The Court held that no damage has been caused because of the defendant has added no extra-damage. In our framework, this defendant is a dummy player.

To conclude, characteristic functions of a cooperative game are implicitly taken account by courts when they adjudicate successive injury cases. The notion of potential damage illustrates that courts compare the actual harms of the tortious acts of the tortfeasors and the harms which would have resulted from the action of *some* subsets of tortfeasors. Once available the characteristic function, the issue still holds to know how dividing divisible and indivisible parts among tortfeasors. We now discuss this point.

3.3. Apportionment by causation and by responsibility, the core and the Shapley value

Once damage is divided, the *Restatement* states that the first step is to apportion damage by causation namely: each tortfeasor should pay at least for the damage he would have caused alone and at most for the additional damage he has caused.[34] For most legal theorists, it would be unfair for a tortfeasor to pay for more than what he has caused. This basic principle inspired by corrective justice is accepted as the cornerstone of all acceptable apportionment rules: as asserted by Robertson (2009, p. 1008), following Carpenter (1935), "it has long been regarded as a truism that 'a defendant should never be held liable to a plaintiff for a loss where it appears that his wrong did not contribute to it, and no policy or moral consideration can be strong enough to warrant the imposition of liability in such [a] case'".

As we have demonstrated, the core of a "liability game" is the subset of allocations that verify two conditions. The first one is that the sum of the payments due by each tortfeasor

[32] Obviously, if the boy had not lost his balance, the tortious act would have not been damageable. On the contrary, if the electric company had not been negligent, a less important damage would have occurred. The key-element the factfinder has to know is whether the boy had already lost his balance before grabbing the electric wires or not.

[33] *Douglas Burt & Buchanan Co. V. Texas & Pacific Railway Co.*, 1922, 150 La 1038, 91 So. 503.

[34] This principle is one of the cornerstones of the *Restatement*: "no party should be liable for harm it did not cause" (*Restatement*, Topic 5, §26, *comment* a, see also *comments* h and j). That is why a one step process is unfair: "a court may decide to use a one-step process of apportionment. The factfinder determines the total recoverable damages and then assigns percentages of responsibility to each person who caused some of the damages […]. A problem with a one-step process is that it may result in a party's being held liable for more damages than the party caused. See *comment d*. A party's comparative responsibility is distinct from the magnitude of the injury the party caused" (*Restatement*, Topic 5, §26, *comment* j).

exactly covers the harm suffered by the plaintiff. The second one is the C2 condition: no group of tortfeasors pays more than it has caused ("*non-objectionable adjudications*"). The law implicitly acknowledges the importance of these two restrictions imposed by efficiency and individual rationality to consistently apportion liability. First, the full amount of damage has to be recovered by the victim. Therefore, the sum of all the shares paid by tortfeasors has to be equal to total damage.[35] Second, in litigation where only two or three tortfeasors are causally involved, the legal principle that, at any rates, a tortfeasor should not pay for damage he has not caused leads to apportionment which belongs to the core.[36]

In these cases, legal and economic principles converge: saying that no tortfeasor should pay more than he has caused is a legal translation of the condition C2 in our game. The first tortfeasor in the causal sequence cannot pay less than d_1 ($x_1 \geq d_1$) the damage caused by him alone regardless of what the others tortfeasors have done. The last one cannot pay more than d_n ($x_n \leq d_n$), the aggravated damage he has added at most by his tortious act. The tortfeasor *i* in the liability sequence cannot pay more than c_n - c_{i-1} ($0 \leq x_i \leq d(N) - d(1,\ldots,i\text{-}1)$). An *unobjectionable adjudication* shares these properties. And the *Restatement* and many common law cases follow this requirement. For example, in *Ravo v. Rogatnick*, the Court states that, in case of successive injuries due to medical malpractice, "the initial tortfeasor may well be liable to the plaintiff for the entire damage proximately resulting from his own wrongful acts. The successive tortfeasor, however, is liable only for the separate injury or the aggravation his conduct has caused".[37] On the same basis, Keaton (1984, p. 352), considers "an original wrongdoer may be liable for the additional harm inflicted by the negligent treatment of the victim by a physician, but the physician will not be liable for the original injury".[38]

[35] A plaintiff's total aggregate recovery from all the contributing tortfeasors can never exceed the amount of his actual damages. See *Miller v. Union Pacific R. Co.*, 290 U.S. 227, 236 (1933). We do not deal with punitive damages and we consider that courts are able to calculate the full amount of damage to be paid to the victim. *A priori*, our argument does not depend on the methods actually used by courts to calculate damages except if the calculation leads to non-monotonicity: it could be the case, for example, when a first tortfeasor causes a disease to the victim, following by a second tortfeasor who causes death and compensation for death be less important than compensation for disease.

[36] For *n* larger than 4, the core requires that this condition be extended from individuals to coalitions (see the discussion about condition C1 and C2 *supra*).

[37] *Ravo v. Rogatnick*, 514 N.E.2d 1104 (N.Y. 1987). Or see *Pridham v. Cash & Carry Bldg. Center, Inc.*, 359 A.2d 193 (N.H. 1976), *Prospectus Alpha Navigation Co v. North Pacific Grain Growers, Inc.*, 767 F. 2d 1379 (9th Cir. 1986).

[38] See the *Restatement (second) of Torts*: « it should be noted that there are situations in which the earlier wrongdoer may be liable for the entire damage, while the later one will not. Thus an original tortfeasor may be liable not only for the harm which he has himself inflicted, but also for the additional damages resulting from the negligent treatment of the injury by a physician. The physician, on the other hand, has played no part in causing the original injury, and will be liable only for the additional harm caused by his own negligence in treatment » (*Restatement (second) of Torts*, 16.1.A., §433A, *comment* c).

The previous examples share a common feature: courts follow apportionments that belong to the core as defined in section 2. But, most of times, this apportionment by causation is insufficient to provide a unique apportionment of the damage (the core is non-empty and contains many imputations). One remaining issue is precisely to know how to divide the indivisible components. The second principle proposed by the two-step method – the apportionment by responsibility – is needed: "the court should divide damages by causation and then, *for each component part*, apportion liability by shares of responsibility (emphasis added)".[39] Fault degrees of each tortfeasor are introduced and play the role of relative weights. Dividing indivisible damage by responsibility in the sense of the *Restatement* consists in assigning some weights to each tortfeasor in order to divide the indivisible components. Judge could consider arguments which justify treating unequally the tortfeasors, for example, because their degrees of fault are different.

Let's study one of the numerical examples provided by the *Restatement* to illustrate the two-step process: "Consider a case in which D, the driver of an automobile, is alleged to have negligently driven an automobile into a highway guardrail. An alleged defect in the automobile's door latch causes the passenger door to open and P, the passenger, to be ejected. P suffers serious neurological injuries and sues D and M, the automobile's manufacture […]. The court instructs the jury that it must find what damages P would have suffered if the door had not opened (assuming the jurisdiction recognizes that hypothetical injury as a cognizable injury for purposes of causal division) […]. After making that determination, the jury decides if D and M are legally responsible and assigns percentages of responsibility to them […]. If the jury found that P would have suffered some damages if the door had remained closed, damages are divisible. The jury determines if D was negligent, M's automobile was defective, the negligence caused the entire damages, and the defect caused the enhanced injury. It finds the amount of damages for the enhanced injury and the damages for the entire injury. The jury then assigns percentages of responsibility to D and M for the enhanced injury". [40]

In terms of our model, courts determine the set of tortfeasors $N = \{D,M\}$ and instruct the jury to determine d_1 and d_2 (damage division). This defines the characteristic function v associated to the set of tortfeasors $\{D,M\}$: $v(D) = d_1$, $v(D,M) = d_1 + d_2$ and $v(M) = 0$. The legal issue is to solve the transferable utility game (N, v): the initial damage is entirely paid

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[39] See the *Restatement* Topic 5, §26, *Reporters' note*, *comment* d. This comment criticizes *Alpha navigation* because the additional damage is partly due by the first tortfeasor insofar as without his tortious act the latter damage would have not occurred: "The court stopped with causal division by holding that the defendant was liable for *all* the damage caused by its decision to send the ship away. That is not consistent with the goals of comparative responsibility. The plaintiff's negligence also caused the extra damage; but for the original fire, there would have been no damage".

[40] The *Restatement*, Topic 5, §26, *Reporters' note*, *comment* c.

by D since he is the only cause of this part of the damage and the enhanced injury is shared between the two torfeasors by assigning to each of them a degree of responsibility w_D et w_M knowing that the sum equals 100%. Then the payment due by each tortfeasor from this two-step process is exactly the weighted Shapley value associated to the game (N,v).

All the interest of the weighted Shapley-value is to offer possible compromises consistent with conditions C1 and C2 and with the evaluation by the judge of the responsibility of each. And the weighted Shapley value mathematically distinguishes between causation and responsibility apportionment. Reciprocally, each core-allocation is a particular weighted Shapley-value. As such, it is possible to consider that, as soon as a court chooses a core-allocation, it reveals the degree of responsibility of each tortfeasor.

To finish with, let's compare the Shapley-value axioms with the *Restatement* principles. The core requires two conditions (efficiency and C2 condition); the Shapley value requires four axioms (efficiency, additivity, null player, proportionality) while the *Restatement* states three main principles (full amount of damage recovered by the victim, apportionment by causation, apportionment by responsibility). The symmetric Shapley value requires symmetry to replace proportionality.

Shapley value axioms	**Restatement two steps process**
Efficiency	Full amount of damage recovered
Additivity	Apportionment by causation
Null player	
Symmetry or Proportionality	Apportionment by responsibility

Table 1. Legal principles and the Shapley-value axioms

4. CONCLUSION

By comparing cooperative game theoretical approach with cases and the *Restatement*, we have insisted on the fact that law principles regarding apportionment among multiple tortfeasors could be rationalized on a cooperative game theoretical basis. We would like to conclude with some final comments and with extensions of our analysis. First, we have shown that the deep logic of cooperative game theory – attributing to each player the value he has caused, alone and jointly, and then to evaluate his contribution – captures the legal method proposed in law and used by courts to apportion damage by causation and by responsibility, at least in case of successive injuries. Second, from a "normative" point of view, the stake of our model is to provide some rational basis to discuss and understand the final decision of a judge in sequential liability cases. The most interesting aspect is that apportionment rules can be analyzed on rational and axiomatic grounds. Arbitrariness of judges could be reduced by using such reasoning and clear expectations could be created by applying such rules. To be operational in law, cooperative game theoretical approach requires knowing the value of every coalitions, even the ones which have not occurred. But beyond the monetization, in sequential liability cases, our approach does not require a lot of information (for the case where $n=3$, only three numbers must be obtained, $v(1)$, $v(12)$ and $v(123)$). Because our approach is based on *ex po*st causation, coalitions could be understood as counterfactual states of the world (the state of the world which would have occurred, all things being equal, if one agent had not tortiously acted). The deep conception of causation in the model is the "but-for test" criteria. But, as we have argued, that does not mean that a share by capita is always the necessary rational apportionment. On the contrary, a lot of rational allocations are possible. And Shapley-value is an interesting benchmark that courts could use to better evaluate the contribution of each tortfeasor. Finally, it seems to us that our approach is sufficiently general to be applied to other cases of multiple causation and leaves room open for further developments.

5. APPENDIX: COOPERATIVE GAMES WITH TRANSFERABLE UTILITY

5.1. Characteristic function

A set $N = \{1,...,n\}$ of players, $n \geq 2$, faces the problem of dividing the surplus resulting from their cooperation. The surplus resulting from the cooperation within any coalition of players, including single players, is also known. This defines the *characteristic function v*, a real-valued function on the subsets of *N*, that associates to every coalition $S \subset N$ its *worth* $v(S)$, with the convention that $v(\emptyset) = 0$. The pair (N,v) defines a cooperative game with transferable utility, a *game* for short.

A game (N,v) is *essential* if there is a potential gain to cooperate:

$$v(N) > \sum_{i=1}^{n} v(i) \tag{A1}$$

It is *superadditive* if merging disjoint coalitions can only be beneficial:

$$S \cap T = \emptyset \Rightarrow v(S \cup T) \geq v(S) + v(T)$$

Superadditivity implies that (A1) holds with a weak inequality. Here, the games we consider satisfy a stronger property namely convexity.[41] A game (N,v) is *convex* if the marginal contributions of any player are nondecreasing with respect to set inclusion:

$$i \in S \subset T \Rightarrow v(S) - v(S \backslash i) \leq v(T) - v(T \backslash i) \tag{A2}$$

This is *increasing return to size*. Equivalently, a game (N,v) is convex if *v* is *supermodular*:

$$v(S \cup T) \geq v(S) + v(T) - v(S \cap T) \text{ for all } S,T \subset N$$

Hence convexity implies superadditivity. The simpler example of convex games is the family of *unanimity games* (N,u_T) defined for any given $T \subset N$ by

$$\begin{aligned} u_T(S) &= 1 \quad \text{if } T \subset S \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{A3}$$

It corresponds to the division of a dollar that requires that the players in *T* must all agree: the members of *T* have a veto right.

Given a game (N,v), the problem is to allocate $v(N)$ among the players i.e. identify a vector $x = (x_1,...,x_n)$ such that $x(N) = v(N)$.[42] An *allocation rule* is a mapping $\varphi : (N,v) \to \varphi(N,v)$ that associates to any game (N,v) a vector $(\varphi_1(N,v),...,\varphi_n(N,v))$ such that:

[41] See Shapley (1971).

[42] Efficiency is built in. It requires that an admissible rule should allocate the entire surplus.

$$\sum_{i \in N} \varphi_i(N,v) = v(N)$$

The theory of cooperative games offers a wide variety of solution concepts. Here we shall retain two solution concepts, the *core* and the *Shapley values*. They are different in nature: the core of a game defines a *set* of allocations while the Shapley value is an allocation *rule* that selects a core allocation when applied to a convex game.

5.2. Marginal contributions and the Weber set

Marginal contributions play a central role in surplus allocation, especially when dealing with convex games where they enter directly into the definition of both the core and the Shapley value. A player i is *dummy* in a game (N,v) if he never contributes: $v(S) - v(S \backslash i) = 0$ for all $S \subset N$. Players i and j are substitutable if they contribute equally to the coalition to which they both belong: $i, j \in S \Rightarrow v(S) - v(S \setminus i) = v(S) - v(S \setminus j)$. Superadditivity implies that marginal contributions of a player are bounded below by his individual worth:

$$v(S) - v(S \backslash i) \geq v(i) \text{ for all } i \in S \text{ and all } S \subset N$$

Convexity implies that the marginal contribution of a player is *maximal* at the grand coalition:

$$v(S) - v(S \backslash i) \leq v(N) - v(N \backslash i) \text{ for all } i \in S \text{ and all } S \subset N$$

Let Π_N be the set of all players' permutations. To each permutation $\pi = (i_1, ..., i_n) \in \Pi_N$ we associate the *vector of marginal contributions* $\mu(\pi)$ whose elements are given by:

$$\begin{aligned} &\mu_{i_1}(\pi) = v(i_1) - v(\varnothing) = v(i_1) \\ &\mu_{i_k}(\pi) = v(i_1, ..., i_k) - v(i_1, ..., i_{k-1}) \quad (k = 2, ..., n) \end{aligned} \tag{A4}$$

The last element $\mu_{i_n}(\pi)$ is the marginal contribution of the last player to the grand coalition.

Different permutations may yield the same vector. Hence, the number of *distinct* vectors is typically smaller than n! and a multiplicity can then be attached to each vector. The marginal contribution vectors of to the unanimity game (N, u_N) are given by (1,0,0), (0,1,0) and (0,0,1). Each has multiplicity 2 and gives 1 to the player that is last.

The *Weber set* is the convex hull of the marginal contribution vectors i.e. the smallest convex set containing the marginal contribution vectors. It is therefore equivalently defined as the set of all possible averages of the marginal contribution vectors:

$$W(N,v) = \left\{ x \in \mathbb{R}^n \,\middle|\, x = \sum_{\pi \in \Pi_N} \lambda_\pi \mu(\pi) \text{ for some } \lambda \in \Delta(\Pi_N) \right\}$$

where $\Delta(\Pi_N)$ denotes the set of probability distributions on the set of players' orderings.

5.3. Imputations and core allocations

An imputation is an allocation of the total surplus that is *individually rational* in the sense that no player gets less than his worth. This defines the set of *imputations* of a game (N,v):

$$I(N,v)=\left\{x\in\mathbb{R}^n \,\middle|\, x(N)=v(N) \text{ and } x_i\geq v(i) \text{ for all } i\in N\right\}$$

It is easily seen that marginal contribution vectors are imputations:

$$\sum_{i=1}^{n}\mu_i(\pi)=v(N) \text{ and } \mu_i(\pi)\geq v(i) \text{ for all } i\in N$$

The imputation set is a *regular simplex* of dimension $n-1$ if the game is essential. Its centre of gravity defines the *equal surplus* allocation rule:

$$ES_i(N,v)=v(i)+\frac{1}{n}\left(v(N)-\sum_{j\in N}v(j)\right) \qquad \text{(A5)}$$

Each player receives his individual worth and what remains is then uniformly allocated.

The *core* of a game is a concept introduced by Gillies (1953). It is the set of imputations that *no coalition can improve upon*:

$$C(N,v)=\left\{x\in\mathbb{R}^n \,\middle|\, x(N)=v(N) \text{ and } x(S)\geq v(S) \text{ for all } S\subset N\right\} \qquad \text{(A6)}$$

Indeed**,** if $x(S)<v(S)$ for some coalition S, $v(S)$ can be redistributed among the members of S in such a way that each of them gets more. This is the usual interpretation of the core as a set of "stable" allocations. The core of the unanimity game (N,u_T) consists of the non-negative vectors that allocate 1 to the members of T.

Being defined by linear inequalities, the core is a *polyhedron* whose dimension is at most n–1, possibly empty. The core of a convex game is the polyhedron whose vertices are precisely the marginal contribution vectors.[43] Consequently, by their definitions, the core and the Weber set coincide when applied to a convex game:

$$(N,v) \text{ convex } \Rightarrow C(N,v)=W(N,v) \qquad \text{(A7)}$$

Using the identity $x(N)=v(N)$, Core allocations satisfy the following *equivalent* inequalities:

$$x(S)\leq v(N)-v(N\backslash S) \text{ for all } S\subset N \qquad \text{(A8)}$$

[43] See Shapley (1971).

i.e. no coalition can expect to get more than its marginal contribution to *N*. In particular,

$$x_i \leq v(N) - v(N \setminus i) \text{ for all } i \in N \tag{A9}$$

i.e. no player can expect to get more than his/her marginal contribution to the grand coalition.

5.4. The symmetric Shapley value

The Shapley value of a game (N, v) is the allocation rule defined by the average marginal contribution vector:

$$SV_i(N,V) = \frac{1}{n!} \sum_{\pi \in \Sigma_N} \mu_i(\pi) \quad i = 1,\ldots,n \tag{A10}$$

Shapley (1953, 1981) has shown that the above rule is the *unique* allocation rule that satisfy the following three axioms on the set of superadditive games.

Symmetry:

if *i* and *j* are substitutable in a game (N,v) then $\varphi_i(N,v) = \varphi_j(N,v)$.

Dummy player:

if player *i* is dummy in a game (N,v) then $\varphi_i(N,v) = 0$.

Additivity:

$$\varphi(N, v+w) = \varphi(N,v) + \varphi(N,w)$$

Symmetry is nothing but "*equal treatment of equals*", a requirement that one finds all across the literature on fair division. Here only the dummy player property is needed to cover the unequal treatment of unequal's. Additivity is an independence property that gives the Shapley value its linear structure.[44] In the unanimity game (N, u_T) the players in *T* are substitutable. and the players outside *T* are dummy. Hence, the Shapley value is given by

$$\begin{aligned} SV_i(N,u) &= \frac{1}{t} \quad \text{for all } i \in T \\ &= 0 \quad \text{for all } i \notin T \end{aligned}$$

Alternative axiomatizations have been proposed.[45] The most interesting one is due to Young (1985) who has introduced the following property:

Marginalism:

$$v(S) - v(S \setminus i) = w(S) - w(S \setminus i) \text{ for all } S \subset N \;\Rightarrow\; \varphi_i(N,v) = \varphi_i(N,w)$$

[44] In Shapley's 1953 paper, a game is a set function defined on a *universe* of players so that efficiency and the dummy player property are combined into a single axiom.

[45] See for instance Moulin (1988, 1995).

This is a strong *independence* requirement. It says that only marginal contributions enter into account for the evaluation of what is allocated to a player, independently of the rest of the game. This property has deep consequences. Indeed Young shows that the Shapley value is the unique rule satisfying marginalism and symmetry. Marginalism replaces additivity as well as the dummy player property.

The allocations defined by the Shapley value are imputations that may not belong to the core. It can however be viewed as a core selection when applied to convex games. Indeed, the core is then the polyhedron whose vertices are the marginal contribution vectors while the Shapley value is the average of the marginal contribution vectors, accounting for multiplicity.

5.5. The weighted Shapley value

Dropping symmetry opens the possibility for equal players to be treated differently. The *asymmetric* version of the value is obtained by introducing exogenous weights to cover asymmetries not included in the characteristic function.[46] For given positive weights $w=(w_1,...,w_n),$ the Shapley value is the average marginal contribution vector

$$SV_i(N,v,w) = \sum_{\pi\in\Pi_N} P_w(\pi)\,\mu_i(\pi) \tag{A11}$$

where

$$P_w(i_1,i_2,...,i_n) = \frac{w_{i_n}}{w_{i_1}+...+w_{i_n}}\frac{w_{i_{n-1}}}{w_{i_1}+...+w_{i_{n-1}}}\cdots\frac{w_{i_2}}{w_{i_1}+w_{i_2}}$$

is the probability distribution induced by w on Π_N. With normalized weights, $w(N)=1$ and w_i is the probability that player i be *last* in an arbitrary ordering. The symmetric value corresponds to equal weights and the normalized weight w_i defines the share of player i in the *unanimity game*:

Proportionality:

$$SV_i(N,u_N,w) = w_i$$

We denote by $WS(N,v)$ the set of *all* weighted values obtained by considering all limits for some but not all weights tending to zero. By definition, the weighted Shapley value is a random order value. Hence, the set of weighted values is a subset of the Weber set:

$$WS(N,v) \subset W(N,v)$$

[46] It was introduced by Shapley (1953). See Kalai and Samet (1987) for a complete characterization. See Dehez (2011) for an axiomatization in a cost-sharing framework.

Monderer, Samet and Shapley (1992) have shown that the core is a subset of the set of weighted values. This is a remarkable result that holds for all game. Hence, we have the following sequence of inclusions:

$$C(N,v) \subset WS(N,v) \subset W(N,v)$$

But we know that the core and the Weber set coincide for convex games (A6). Consequently, the core coincides with the set of weighted Shapley value for convex games:

$$(N,v) \text{ convex } \Rightarrow C(N,v) = WS(N,v) \tag{A12}$$

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